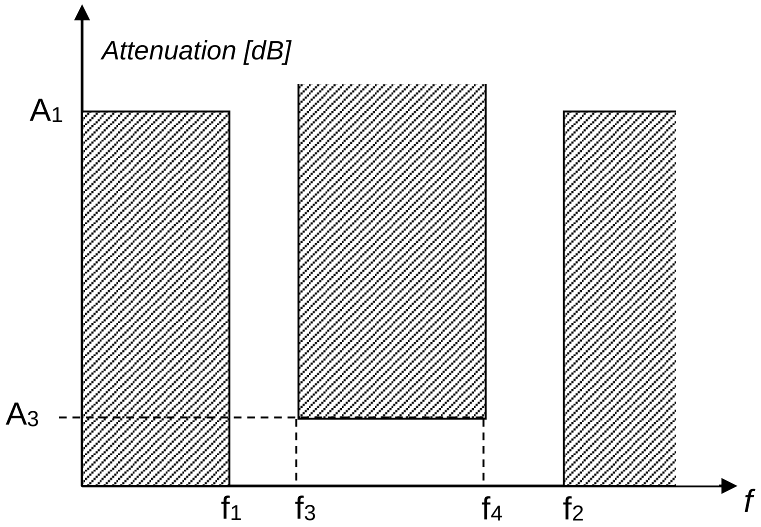
HF&VHF Circuits
and Techniques I

Serie 7 on Chapter 4
Determination of the order N and of the parameter ε of a Chebyshev filter

The specifications of the band-pass filter with respect to the attenuation are given below :



Stop-bands: A_1 = 30 dB, f_1 = 440 kHz, f_2 = 475 kHz
Pass-band : A_3 = 1 dB, f_3 = 450 kHz, f_4 = 460 kHz

We have at our disposal the library of low-pass Chebyshev filters such that the cut-off frequency (the end of the pass-band) is equal to:
$\frac{1}{2\pi}[Hz]$

a)
Modify the specifications of the band-pass filter such that the transformation:
low-pass filter ←→ band-pass filter
can be applied.

Answer:

According to relation 4.6 and to page 4-10 of the course, we have with respect to the pass-band:

$$f_3 \cdot f_4 = f_0^2 \Rightarrow f_0 = \sqrt{450kHz \cdot 460kHz} = 454.972527kHz$$

In order to apply the *low-pass filter* ⇔ *band-pass filter*, we also need to have $f_1 \cdot f_2 = f_0^2$. This is required for applying the transformation because geometrically, f_0 is the symmetry axis of the transformation (see page 4-9 of the course).

In our case :

$f_1 \cdot f_2 = f'^2_0 \Rightarrow f'_0 = \sqrt{440kHz \cdot 475kHz} = 457.1652kHz.$

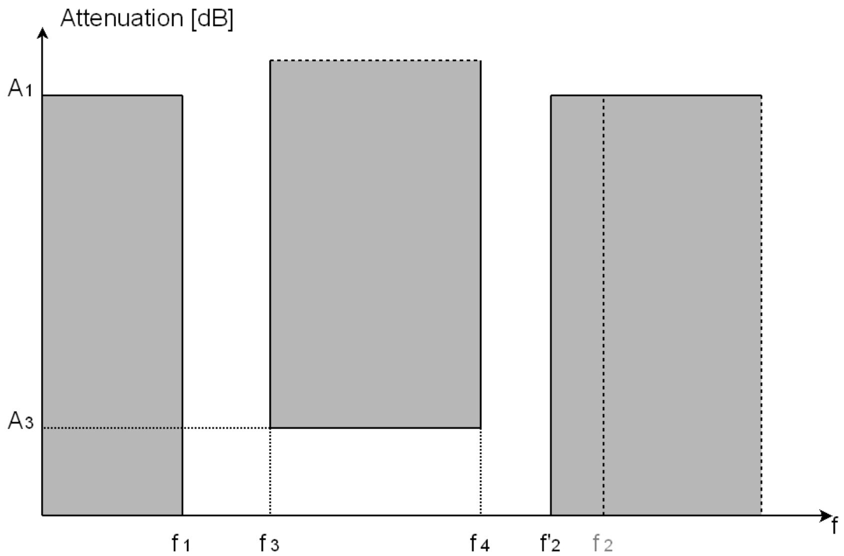
The two frequencies f_0 and f'_0 are not equal, thus we cannot apply the transformation on the given values. We have to modify the specifications in order to have both f_0 and f'_0 to be the same without decreasing the given constraints.

We choose to keep $f_1 = 440kHz$ unchanged and we calculate the new value of f_2 as follows :

$f'_2 = \frac{f_0^2}{f_1} = \frac{(454{,}972527kHz)^2}{440kHz} = 470.4545kHz.$

This choice was made because we have $f_1 \cdot f_2 > f_3 \cdot f_4$.

We thus obtain the following new specifications :



Stop-bands: $A_1 = 30$ dB, $f_1 = 440$ kHz, **$f'_2 = 470.4545$ kHz**
Pass-band : $A_3 = 1$ dB, $f_3 = 450$ kHz, $f_4 = 460$ kHz

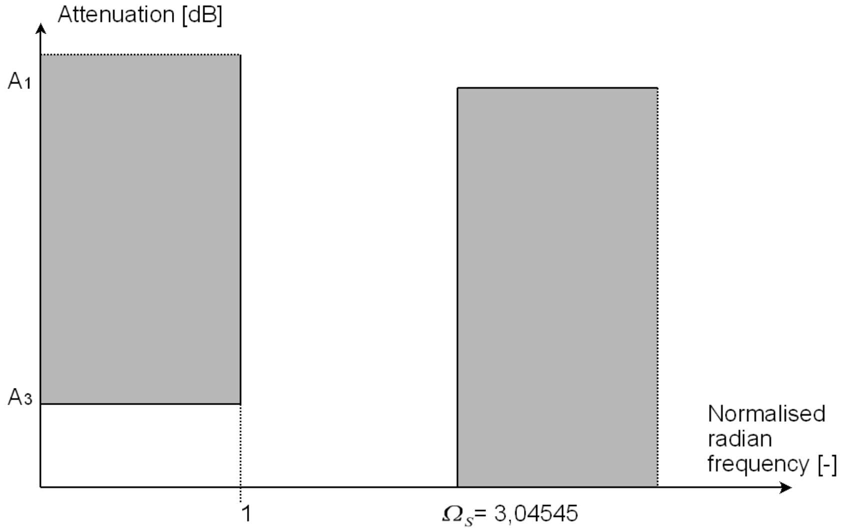
b)
Draw the specifications of the low-pass filter linked to the ones of the band-pass filter determined in question a) such that its cut-off frequency (the end of the pass-band) is equal to: $\frac{1}{2\pi}[Hz]$

Answer:

We use specifications obtained in 1). According to relation 4.9 in page 4-10 of the course, we have the following relation :

$$\Omega_S = \frac{2\pi \cdot f'_2 - 2\pi \cdot f_1}{2\pi \cdot f_4 - 2\pi \cdot f_3} = \frac{f'_2 - f_1}{f_4 - f_3} = \frac{470{,}4545kHz - 440kHz}{460kHz - 450kHz} = 3.04545$$

This relation gives directly the normalized radian frequency of the stop band of the corresponding low-pass filter. Note that a normalized radian frequency has no unit since it represents a ratio between two radian frequencies. We thus obtain the following specifications for the low-pass filter :



Pass-band : $A_3 = 1$ dB, $\Omega_p = 1$.
Stop-band : $A_1 = 30$ dB, $\Omega_s = 3.04545$.

c)
Determine the order N and the parameter ε of the low-pass Chebyshev filter determined in question b).

<u>Answer</u>:

From relation 4.17, we have :

$A_1 = 10\log[1 + \varepsilon^2 C_N^2(\Omega_s)]$ (1) at the beginning of the stop-band
$A_3 = 10\log[1 + \varepsilon^2 C_N^2(\Omega_p = 1)] = 10\log[1 + \varepsilon^2]$ (2) at the end of the pass-band

From (2), we have :

$$\varepsilon^2 = 10^{\frac{A_3}{10}} - 1$$

and thus :

$$\varepsilon = \sqrt{10^{\frac{A_3}{10}} - 1} = \sqrt{10^{\frac{1}{10}} - 1} \approx 0.50885$$

We calculate N from (1) as follows : since $1 < \Omega_s$ because of the normalization and the fact that the stop band occurs at a higher frequency than the pass band frequency (it is a low pass filter), we have :

$$C_N(\Omega_s) = \cosh(N \cdot \operatorname{arcosh}(\Omega_s)).$$

Since we are allowed to have stronger specifications, the filter function can go beyond the limit $A_s = A_1 = 30\ dB$ and we can thus write (1) as :

$$A_s \leq 10\log[1 + \varepsilon^2 \cdot \cosh^2(N \cdot \operatorname{arcosh}(\Omega_s))]$$

$$\Rightarrow \frac{10^{\frac{A_1}{10}} - 1}{\varepsilon^2} \leq \cosh^2(N \cdot \operatorname{arcosh}(\Omega_s))$$

$$\Rightarrow \frac{\sqrt{10^{\frac{A_1}{10}} - 1}}{\varepsilon} \leq \cosh(N \cdot \operatorname{arcosh}(\Omega_s))$$

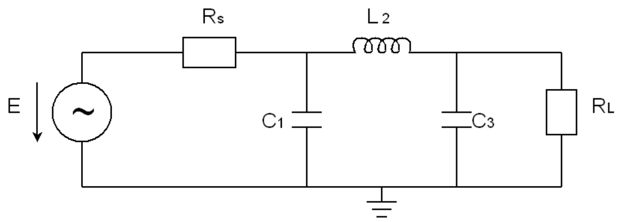
$$\Rightarrow \operatorname{arcosh}\frac{\sqrt{10^{\frac{A_1}{10}} - 1}}{\varepsilon} \leq N \cdot \operatorname{arcosh}(\Omega_s)$$

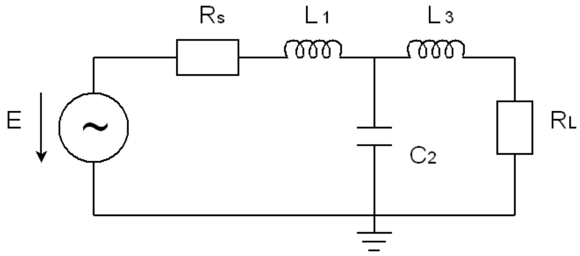
$$\Rightarrow N \geq \frac{\operatorname{arcosh} \frac{\sqrt{10^{\frac{A_1}{10}} - 1}}{\varepsilon}}{\operatorname{arcosh}(\Omega_s)}$$

In addition, the order of the filter has to be an integer since it corresponds to the number of elements of the filter, so it will be rounded to the highest integer in order to reach the specifications. We have :

$$= \left\lceil \frac{acosh \frac{\sqrt{10^{\frac{A_1}{10}} - 1}}{\varepsilon}}{acosh(\Omega_s)} \right\rceil = \left\lceil \frac{acosh \frac{\sqrt{10^{\frac{30}{10}} - 1}}{0.50885}}{acosh(3{,}04545)} \right\rceil = \lceil 2.711\ldots \rceil = 3$$

d)
Draw the two possible topologies of the low pass-filter.





Remark:

- We can use the first or the second topology as there will be then the transformation from this loss-pass filter to the band-pass filter to find the values of the components which will fulfill the specifications of the band-pass filter determined in question a).
- The first topology and the second topology of the low-pass filter will be transformed in a band-pass filter which contains 3 capacitors and 3 inductors.